APPLICATIONS OF FUZZY LOGIC IN OPERATIONAL METEOROLOGY

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The topic of Fuzzy Logic as a decision-making technique is introduced. It is recommended that applications of this technique could be effectively applied in the area of operational meteorology. An example of such an application, the forecast of the probability of formation of radiation fog, is discussed and examples of the method are presented. Other possible meteorological applications are suggested. Additionally, a software package which aids in the development of such applications is briefly described.

1. Introduction

Even as rigorous numerical modelling of meteorological processes continues to improve in both spatial and temporal resolution, there will likely continue to be processes that elude explicit analytic solutions. Physical processes not yet well understood or those beyond the resolution of the models still need alternative methods for their analysis and subsequent prognosis. Mesoscale processes such as fog, stratus and convection have always been difficult to forecast. Most experienced forecasters will quickly suggest that experience is the best tool for forecasting such events. With the rapidly evolving technologies in the field of meteorology, it is desirable to merge the experience of many forecasters with algorithms that may aid in difficult forecasting situations. Cognitive computing has been an emergent set of problem solving algorithms that attempt to imitate natural problem solving techniques. One such method is called Fuzzy Logic.

Fuzzy Logic is a simple yet very powerful problem solving technique with extensive applicability. It is currently used in the fields of business, systems control, electronics and traffic engineering. The technique can be used to generate solutions to problems based on "vague, ambiguous, qualitative, incomplete or imprecise information."¹ There are many weather forecasting difficulties that fit the
previous description - where rigorous, analytical solutions do not exist. Fuzzy Logic is an extension of Fuzzy set theory that was developed approximately 30 years ago. The paradoxical name requires a short explanation. The intent of Fuzzy set theory was to alleviate problems associated with traditional binary logic, where statements are exclusively true or false. Fuzzy Logic allows something to be partially true and partially false. A simple example follows: Is a man who stands 170 centimetres (~5'6") considered to be tall? Traditionally we must define a threshold over which a man of a certain height is considered a member of the tall set and under which he is not. Fuzzy Logic allows one to speak of a 170 cm man as both a member of the tall set and the medium set, and possibly even the short set. He may be considered to a larger degree a member of the medium set than he is of the tall set. A man who stands 190 centimetres will be to a higher degree a member of the tall set. If a problem suggests there is some consequence related to the height of a tall man, then the consequence can be applied or inferred in relation to his degree of membership in the tall set.

Those are many instances within weather forecasting which cannot be easily handled by methods other than statistical methods, "perfect prog" methods, pattern recognition, and other less analytic methods. In some cases, decision trees or flow charts have been developed. However, these are based on traditional logic. For instance in forecasting lake enhanced snow flurries over the Great Lakes, one rule-of-thumb which is considered is the length of fetch of the air flow over open water.² A threshold value of 80 nautical miles is considered highly favourable. Statistical studies or qualitative physics may have suggested this "critical" value of 80 nautical miles. With all other factors being equal, a fetch of only 70 nautical miles would probably still contribute to the development of snow squalls, perhaps however to a lesser extent. Fuzzy Logic can account for the effect upon the output of a system due to various input values, without the need for a definite threshold value. The consequence (the development of lake enhanced snow) is physically related to several antecedents, of which fetch is only one. Up to a physical limit the consequence in this example is proportional to the value of one of the antecedents.
2. A Fuzzy System: Radiation Fog

The meteorological phenomenon chosen here to demonstrate this method is that of the occurrence of radiation fog. This problem was chosen since it is a common event in most regions in Canada. Several techniques have been developed and used over the years to aid in the forecasting of the onset of radiation fog. Specifically, methods developed by Jacobs and Bartham have been taught for years. Although no analytic method has yet been discovered, the basic physical principles involved in fog forecasting are thought to be well understood.

2.1. System Inputs and Fuzzy Sets

Within the context of the current example of fog forecasting, general terms associated with Fuzzy Logic will be introduced. In general, a problem to be solved is referred to as a system. System inputs are those physical variables that are thought to completely determine the solution(s) to the problem, or system outputs. In the current example the system output is the probability of the occurrence of radiation fog within the next (approximately) six hours. System inputs to be used in this case are the values of dew point, dew point spread, the rate of change of the spread, the wind speed and the sky coverage.

These system inputs are readily available, and their values are typically considered when a forecaster decides if or when radiation fog will develop. It is understood at this point there are other physical variables that forecasters may consider important in the formation of radiation fog. For example in many locations a preferential wind direction is conducive to formation, such as a flow off a body of water or from a source of condensation nuclei such as an industrial area. However, the purpose here is to demonstrate the method in a general case. The Appendix will outline a method by which users can develop their own Fuzzy systems. Source code can be provided which simplifies this task.
The system inputs are categorized into physically significant domains called fuzzy sets. Fuzzy sets are simply qualitative descriptions of the chosen domains of the inputs, each of which is thought to have a specific effect on the output. Table 1 shows the fuzzy sets used in this example.

<table>
<thead>
<tr>
<th>Dewpoint</th>
<th>Spread</th>
<th>Rate</th>
<th>Wind</th>
<th>Sky</th>
</tr>
</thead>
<tbody>
<tr>
<td>dry</td>
<td>unsaturated</td>
<td>drying</td>
<td>too light</td>
<td>cloudy</td>
</tr>
<tr>
<td>moderate</td>
<td>saturated</td>
<td>saturating</td>
<td>excellent</td>
<td>clear</td>
</tr>
<tr>
<td>moist</td>
<td>very saturated</td>
<td></td>
<td>too strong</td>
<td></td>
</tr>
<tr>
<td>very moist</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Table 1. Fuzzy Sets in the Radiation Fog Example.

The interpretation of the qualitative description of these fuzzy sets, and the physical effect on the probability of occurrence of radiation fog should be obvious. The value of the current dewpoint is categorized into one of four fuzzy sets: dry, moderate, moist and very moist. Similarly, the value of the dewpoint spread is described as unsaturated, saturated or very saturated. The rate of change of the dewpoint spread (°C/hour) is only characterized by one of two categories, whether the atmosphere is drying (a positive rate) or shows a saturating trend (a negative rate). The wind's effect upon the probability of fog formation is described as either too strong or too light, both unfavourable conditions, or being in the range that is favourable for fog formation. Similarly the sky conditions are classified into either cloudy or clear.

The determination of the fuzzy sets is derived solely from experience. A developer of a fuzzy system must judiciously choose these categories such that reasonable systems outputs will be obtained. This is the critical process of tuning, or calibrating, the system. This will be discussed in detail later. It is here where the experience of the system developer becomes very important.
The fuzzy sets are quantitatively defined by *membership functions*. These functions are typically very simple functions that cover a specified domain of the value of the system input. The functions are generally trapezoids, although simpler functions such as triangles and rectangles and even delta functions are often used. The fuzzy sets in this example are shown in Figures 1 through 5.

**Figure 1.** Membership functions associated with the dewpoint are referred to as "dry", "moderate", "moist" and "very moist."
Figure 2. Membership functions associated with the dewpoint spread are referred to as "very saturated," "saturated" and "unsaturated."

Figure 3. Membership functions associated with the rate of change of the dewpoint spread are "becoming saturated" and "drying."
Each value of system input will belong to at least one fuzzy set and very likely more than one fuzzy set. This is possible because during construction the adjacent fuzzy sets are made to overlap. A rule of thumb is to ensure that the sets overlap by approximately 25 percent. This is the foundation of

**Figure 4.** Membership functions associated with the wind speed are referred to as "too light", "excellent" and "too strong."

**Figure 5.** Membership functions associated with the sky condition are referred to as "clear" and "cloudy."
the technique of Fuzzy Logic. For example in Figure 1 a dewpoint value as indicated of 6°C is seen to be both a member of the "moderate" and the "moist" sets. The membership function associated with each fuzzy set provides a quantitative value of the degree of membership of the input into each fuzzy set. In Figure 1 a value of 6°C is a 48% member of the "moderate" set, a 72% member of the "moist" set and a 0% member of the remaining sets. Note the entire physical domain of each system input must be described by fuzzy sets. The extremes of the value of the dewpoint, dry and very moist, are described by membership functions that simply are horizontal lines with values of 100%. In some cases the membership functions describe nonphysical values of inputs such as a negative dewpoint spread. It is noted this does not have any adverse effect upon the algorithm.

The degrees of membership of the system inputs are evaluated as discussed above. The determination of the system outputs (probability of fog formation) follows from the evaluation of a set of predefined rules. The strength of a rule is derived from the corresponding degrees of membership of the system inputs. Since an input can be member of multiple fuzzy sets, then different rules involving these sets can be applied. The higher degrees of membership result in corresponding rules which have more strength in the final evaluation process.

2.2. The Rule Base

The rule base is a set of rules of the If-Then form. The If portion of a rule refers to the degree of membership in one of the fuzzy sets. The Then portion refers to the consequence, or the associated system output fuzzy set (to be discussed later). For example, one rule could be stated:

\[
\text{If (dry \& unsaturated \& drying \& too light \& cloudy) Then (low probability of fog)}
\]

The rules must be constructed for each system and must rely on the experience of the forecaster/developer. The total number of rules is the product of the number of fuzzy sets characterizing the system. In other words, the number of rules equals all possible permutations of categorized system inputs. Here there are four sets associated with the dewpoint, three with the dewpoint spread, two with the rate of change of the spread, three with the wind speed and two with the sky condition. The total number of rules that completely define the set then is \(4 \times 3 \times 2 \times 3 \times 2 = 144\).
It is easy to see how the number of rules rapidly expands with each system input and related fuzzy sets. The Appendix describes a local software package that simplifies the handling of the rules development.

The strength of a rule is the value of its least true antecedent, or \( I_f \) portion, which is simply the degree of membership of each system input in the corresponding fuzzy set(s). More than one rule can lead to the same consequence. In this case, the rule with the highest strength is used. An example will be presented in the next section.

2.3. System Outputs

The system outputs are also defined by membership functions similar to the inputs. The sets for the present example are given in Figure 6.

![Figure 6. Output membership functions associated with the probability of fog formation. Possible probabilities include "very low", "low", "medium", "high" and "very high."](image)

The output membership functions aid in determining a final value of the system output. The process is essentially the inverse of the evaluation of the degrees of membership. The methodology employed is referred to as the centre-of-gravity method in which the area of all applicable output
sets, limited in height by the applied rule strength, are evaluated and a weighted mean value of the output is calculated. Again, the example in the next section will clarify this.

3. Examples

This section will demonstrate an explicit example of the technique as well as indicate some preliminary results using real data. The purpose of the first section is to describe the algorithm in detail. The second section will indicate the promise of the technique as applied to the current forecast problem.

3.1. The Algorithm in Detail

For the sake of demonstration, assume the following data:

- dewpoint = 6°C
- dewpoint spread = 1°C
- rate of change of dewpoint spread = -1°C/hour
- wind speed = 7 knots
- sky coverage = 2 tenths

The first step is to determine the degrees of membership in each of the fuzzy sets for each system input. Refer to Figures 1 through 5 for the assignment of membership values. As indicated before, this value of the dewpoint allows it to be a 72% member of the "moist" set and a 48% member of the "moderate" set. It is a 0% member of the other sets shown. A spread of 1°C indicates a 66% member of the "saturated" set and a 50% member of the very saturated set. A rate of -1°C/hour suggests a 100% member of the "saturating" set and a 33% member of the "drying" set. Similarly, the wind speed of 7 knots gives a 100% membership to the "excellent" set and 60% membership to the "too light" set. A sky coverage of 2 tenths is a 100% member of the "clear" set and a 0% member of the cloudy set.
In this example only 16 rules of the entire set of 144 will have non-zero values, or strengths. This is because of the membership assignment above: the dewpoint, spread, rate and wind each involve two fuzzy sets while the sky coverage only involves one set ($2 \times 2 \times 2 \times 1 = 16$).

The antecedents of all applicable rules along with the predefined corresponding output sets are:

1. If (moderate & saturated & drying & excellent & clear) Then (low probability of fog)
2. If (moderate & saturated & drying & too light & clear) Then (low probability of fog)
3. If (moderate & saturated & saturating & excellent & clear) Then (high probability of fog)
4. If (moderate & saturated & saturating & too light & clear) Then (medium probability of fog)
5. If (moderate & very saturated & drying & excellent & clear) Then (medium probability of fog)
6. If (moderate & very saturated & drying & too light & clear) Then (medium probability of fog)
7. If (moderate & very saturated & saturating & excellent & clear) Then (very high probability of fog)
8. If (moderate & very saturated & saturating & too light & clear) Then (high probability of fog)
9. If (moist & saturated & drying & excellent & clear) Then (medium probability of fog)
10. If (moist & saturated & drying & too light & clear) Then (medium probability of fog)
11. If (moist & saturated & saturating & excellent & clear) Then (very high probability of fog)
12. If (moist & saturated & saturating & too light & clear) Then (very high probability of fog)
13. If (moist & very saturated & drying & excellent & clear) Then (high probability of fog)
14. If (moist & very saturated & drying & too light & clear) Then (high probability of fog)
15. If (moist & very saturated & saturating & excellent & clear) Then (very high probability of fog)
16. If (moist & very saturated & saturating & too light & clear) Then (very high probability of fog)

The next step is to determine the rule strengths. Recall the strength of a rule is the value of its weakest or least valued antecedent. In the first applicable rule above the value of its antecedents are, in order: 48% (moderate dewpoint), 66% (saturated spread), 33% (drying rate), 100% (excellent wind speed) and 100% (clear skies). The rule strength is then 33. In a similar fashion, the other rule strengths are evaluated and are listed below.
The next step is to derive a system output, a probability of fog formation, from the applicable rules. Note that several sets of rules above share the same consequence, or system output. The value of the output is assigned the value of the "most true", or strongest, rule. To illustrate, rules one and two above have as their consequence a low probability of fog formation. Table 2 shows that rules one and two both have a strength of 33, and consequently then the limiting value associated with a low probability of fog is assigned 33. It will be seen next how this limiting value is employed. Table 3 summarizes the limiting values.

<table>
<thead>
<tr>
<th>Rule</th>
<th>Value</th>
<th>Rule</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>33</td>
<td>9</td>
<td>33</td>
</tr>
<tr>
<td>2</td>
<td>33</td>
<td>10</td>
<td>33</td>
</tr>
<tr>
<td>3</td>
<td>48</td>
<td>11</td>
<td>66</td>
</tr>
<tr>
<td>4</td>
<td>48</td>
<td>12</td>
<td>60</td>
</tr>
<tr>
<td>5</td>
<td>33</td>
<td>13</td>
<td>33</td>
</tr>
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<td>6</td>
<td>33</td>
<td>14</td>
<td>33</td>
</tr>
<tr>
<td>7</td>
<td>48</td>
<td>15</td>
<td>50</td>
</tr>
<tr>
<td>8</td>
<td>48</td>
<td>16</td>
<td>50</td>
</tr>
</tbody>
</table>

**Table 2.** Rule Strengths in the current example.

<table>
<thead>
<tr>
<th>very low</th>
<th>low</th>
<th>medium</th>
<th>high</th>
<th>very high</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>33</td>
<td>48</td>
<td>48</td>
<td>66</td>
</tr>
</tbody>
</table>

**Table 3.** Limiting values for each of the system output fuzzy sets.
Figure 6 indicates how these limiting values for each output fuzzy set are used. An individual output membership function is restricted in height by its corresponding limiting value. Then the remaining area of the membership function is calculated. The final system output is then calculated as the weighted average of the centroid of each membership function, with the area of the enclosed set used as the weighting factor. Table 4 summarizes the current values.

<table>
<thead>
<tr>
<th>Output fuzzy set</th>
<th>very low</th>
<th>low</th>
<th>medium</th>
<th>high</th>
<th>very high</th>
</tr>
</thead>
<tbody>
<tr>
<td>centroid</td>
<td>15</td>
<td>30</td>
<td>50</td>
<td>70</td>
<td>85</td>
</tr>
<tr>
<td>area</td>
<td>0</td>
<td>825</td>
<td>1536</td>
<td>1056</td>
<td>1254</td>
</tr>
</tbody>
</table>

**Table 4.** Centroids and weighting areas of output fuzzy sets.

The final system output is then simply calculated as:

\[ S_i(centroid)_i \times (area)_i / S_i(area)_i = 60.3 \]

This value is interpreted as a probability of radiation fog formation of 60%.

### 3.2. Preliminary Results

Briefly presented in this section are some recent examples of the technique as applied to real data. While not in operation at the time of these examples, the method nonetheless demonstrates promise.
The first example involves Regina, Saskatchewan on October 11, 1993. The actual observations are presented along with the associated probability of fog formation by radiational cooling.

YQR SA 1000 CLR 15 134/1/-1/3209/988/ $\sim$6400?
YQR SA 1100 CLR 15 145/0/-2/3007/991/ $\sim$1600?
YQR SA 1200 250 -SCT 15 156/-1/-2/3208/993/CI 2023 $\sim$6702?
YQR SA 1300 140 -SCT 250 -SCT 15 158/-2/-3/3308/994/AC1/CI F PTCHS ?1014?
YQR SA 1400 140 -SCT 250 -BKN 4F 165/-2/-2/3306/996/SC1/AC1 $\sim$6318?
YQR SP 1440 5 -SCT 250 -OVC 4F 3407 ST1CI

{48%}
{47%}
{57%}
{53%}
{59%}

This series of actuals shows a typical situation which results in radiation fog. Generally clear skies combined with sufficient moisture and a saturating trend and light winds eventually results in fog formation. In fact, the 1500 UTC actual totally obscured conditions W1 X 1/2F. The indicated probability of fog eventually converges on a 59% probability which corresponds to a medium to high value as defined by the current set of output membership functions.

The second example uses data from Moose Jaw, Saskatchewan on October 8, 1993.

YMJ SA 0400 CLR 15 186/0/-2/1604/001/ $\sim$0500?
YMJ SA 0500 CLR 15 186/0/-1/1503/001/ $\sim$0100?
YMJ SA 0600 CLR 15 184/-1/-2/1707/999/ FROIN 8003 ?9900?
YMJ SA 0700 CLR 15 182/-1/-2/1704/999/ FROIN ?3100?
YMJ SA 0800 CLR 4F 184/-3/-4/0802/999/ FROIN ?9800?

{45%}
{49%}
{54%}
{52%}

Once again the method indicates a higher probability of fog formation as the situation develops. A special observation was sent from Moose Jaw at 0831 UTC, indicating the deterioration of conditions to -X 1/8F.

Unfortunately the time allocated for the development and implementation of this project was quite short and only a few sufficient fog events were available to test the system. However the above results show promise and it is hoped that additional time can be allocated to further develop the system. It is suggested at this point that seasonally dependent systems may be constructed. The current system may tend to underforecast the probability of fog at near freezing temperatures. This is undoubtedly a reflection of the author's inexperience with this phenomenon.
4. Discussion

The previous example demonstrates the use of a Fuzzy system in operational meteorology. It also suggests the experience of the developer is very important. Careful construction of the membership functions as well as the rule base is necessary. These parameters will likely have to be patiently calibrated during development so reasonable results are obtained. Perhaps an extensive data search could be employed in numerous cases where radiation fog did occur, and the limits of the data could be used to deduce suitable fuzzy sets and rules.

The effect of the shape of membership functions upon the solution is very important. Broader input membership functions, those with an extended domain with membership of 100%, have a larger weighting during the rule evaluation. This will be reflected in the final solution. Narrow output membership functions, the consequences of the rules, map the solution into a more restricted domain but the associated weighting by area is less.

This decision-making method is very flexible and could be applied in a variety of forecasting situations. Other mesoscale situations for which fuzzy systems could be developed include stratus formation and dissipation, (non-frontal) thunderstorm development, snow squall situations and ice crystal formation.

This procedure could be easily automated and used in such projects as Scribe, FPA and the FTAUTO program. The code which accompanies this document is easily adaptable to a number of platforms. Very little computing resources are required to implement this type of system. A number of such systems could be developed and inserted into the cron table of any UNIX system and could be run hourly. Alerts messages could be displayed if a system recognized the possibility of the development of some meteorological element.

Appendix

The section briefly describes the software that can be used to develop a Fuzzy system similar to that presented in this paper. It is reiterated the example here is primary for illustrative purposes. Users
are encouraged to alter the parameters to suit their own needs. Mesoscale processes are generally very dependent on the locality.

The motivation for this work followed from an article in the February, 1993 edition of the *Dr. Dobb's Journal*. The focus of this issue was techniques of Cognitive Computing, Fuzzy Logic being introduced as one such example. The article included a code fragment demonstrating the technique.

This "inference engine" was used in this work with only minor modifications. However local software was produced to work in association with that available in the original article. A program, GENFUZ, was developed to aid in the production and development of an operable Fuzzy system. GENFUZ generates two files, constructed from the user's input, that can be combined with the original code to produce an executable system. GENFUZ will take input (ie. user defined system inputs and outputs and their associated membership functions) from the terminal or alternatively from an existing text file. This input will consist of the definitions of the system inputs and outputs and their corresponding membership functions.

If the input is originally by terminal (for a new project), the user can save the input to a text file for later use. In this case, GENFUZ will automatically generate a first guess at the rules assignment by permuting over all possible fuzzy sets as described in the paper. The user will subsequently need to edit this file to alter the rules to ensure appropriate consequences are assigned to antecedents. The previously entered membership functions can also be edited as needed. During subsequent development GENFUZ can take the input from a previously saved text file.

GENFUZ produces a file defining all variables to be used by the inference engine. The second file generated initializes all the data structures to be used in the project. These two files can be put into a *make file* with the original code to be compiled and linked. The code has been tested on stand-alone PC's as well as the HP 9000 UNIX system. The code is written in ANSI standard C with the intention of being completely portable to different operating systems. As a result, the user interface is very simplistic (text mode) to ensure portability. Enthusiastic users may wish to locally develop their own interface if desired.
GENFUZ is available from the C.F.B. Edmonton Canadian Forces Forecast Centre. Interested parties can contact the office for information on obtaining a copy of the software.

References
2. Forecasting Lake Effect Snow in Ontario, Murphy P., MOC 11 Training Notes - Module 4.3